

RELATIVISTIC GRAVITY WITH A DYNAMICAL PREFERRED FRAME

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*Talk presented by D. Mattingly at CPT01; the Second Meeting on CPT
and Lorentz Symmetry, Bloomington, Indiana, 15-18 Aug. 2001.*

1 Introduction

While general relativity possesses local Lorentz invariance, both canonical quantum gravity ¹ and string theory ² suggest that Lorentz invariance may be broken at high energies. Broken Lorentz invariance has also been postulated as an explanation for astrophysical anomalies such as the missing GZK cutoff ³. Therefore, we seek an effective field theory description of gravity where Lorentz invariance is broken. We will construct a candidate theory and then briefly discuss some of the implications.

2 Construction of the Effective Field Theory

Since we have observational evidence only in a small range of energies relative to the expected Planck energy scale of quantum gravity, it is plausible that boost invariance is broken and as yet unobserved. Rotation invariance has been uniformly explored however, and hence for now we assume rotation symmetry is preserved. A structure that preserves rotation invariance and not boost invariance is that of a preferred frame or “aether” which is mathematically realized by a unit future timelike vector field u^a .

There are many possible Lorentz breaking effects involving u^a . For example, a matter field might possess a modified dispersion relation $\omega^2 = |\vec{k}|^2 - k_0^{-2}|\vec{k}|^4$, where k_0 is a constant (mostly likely of order the Planck energy) that sets the scale of the Lorentz breaking. A Lagrangian that gives the above dispersion for a scalar field ϕ is

$$L_\phi = \frac{1}{2}(\nabla^a \phi \nabla_a \phi + k_0^{-2}(D^2 \phi)^2) \quad (1)$$

where the spatial Laplacian D^2 is defined by

$$D^2\phi = -D^a D_a \phi = -q^{ac} \nabla_c (q_a^b \nabla_b \phi) \quad (2)$$

and the spatial metric q_{ab} is defined by

$$q_{ab} = -g_{ab} + u_a u_b. \quad (3)$$

This type of construction suffices for a Lorentz breaking theory in flat spacetime. However, if we try to couple to gravity by adding this action to the Einstein-Hilbert action, the resulting theory is inconsistent because the fixed vector u^a introduces prior geometry and therefore violates general covariance. (The same would be true for a Lorentz breaking tensor field.) If general covariance is violated in this way then the stress tensor for the matter field will not be conserved, rendering the theory unviable. In order to preserve general covariance it is necessary that u^a become dynamical.

Since we have no underlying theory that tells us what form the u^a kinetic terms might take, we follow the spirit of effective field theory and make a derivative expansion for the u^a Lagrangian. Including all terms with up to two derivatives of u^a and g_{ab} , and neglecting total divergences, we have

$$L_{g,u} = a_0 - a_1 R - a_2 R_{ab} u^a u^b \quad (4)$$

$$-b_1 F_{ab} F^{ab} - b_2 (\nabla_a u_b)(\nabla^a u^b) - b_3 \dot{u}^a \dot{u}_a + \lambda(g_{ab} u^a u^b - 1) \quad (5)$$

where $\dot{u}^a = u^b \nabla_b u^a$ and $F_{ab} = 2\nabla_{[a} u_{b]}$. λ is a Lagrange multiplier such that the unit constraint on u^a is enforced dynamically as an equation of motion. The theory with Lagrangian density $\sqrt{-g}(L_u + L_\phi)$ is generally covariant since it involves no fixed background structures. Such vector-tensor theories have been previously studied by several authors, both with normalized ^{4,5,6} and non-normalized ^{7,8} u^a . The dynamical aether field has gravitational consequences in addition to the non-gravitational effects of the matter-aether coupling. We turn now to a brief discussion of some of these consequences.

3 Observational Consequences

3.1 Field of Static Bodies

The static spherically symmetric solutions for the gravitational field of a body such as the sun are modified by the introduction of u^a . Solar system tests will therefore place constraints on the coefficients in the theory. Unfortunately, neither the general spherically symmetric static solution nor the PPN parameters for this theory are currently known.

The static spherically symmetric solutions for the case where only $a_1, b_1 \neq 0$ have all been found, however ⁶. In this case, the theory is equivalent to a sector of Einstein-Maxwell charged dust theory where the dust has a charge to mass ratio $-1/2\sqrt{b_1}$. There exists a black hole solution of the Reissner-Nordstrom form where the electric charge Q is replaced by the charge of the aether dust that fell into the black hole. This raises the possibility that the general theory may also introduce a new one parameter family of black hole solutions.

3.2 Eötvös Experiments

The coupling of matter to u^a can result non-geodesic free-fall trajectories for particles. This violation of the equivalence principle should be detectable in principle by Eötvös experiments. However, if k_0 is of order the Planck scale current experiments are not sensitive enough to detect this violation if it arises from couplings like those in (1).

3.3 Gravitational Waves

The metric has the usual transverse traceless (TT) modes where the aether is unperturbed, however their speed is generically modified. In addition there are generically three aether-gravity modes. For example, the table below lists the speeds, relevant metric polarization components $\epsilon_{\mu\nu}$, and relevant aether polarization components w^α , for the theory with only $a_1, b_2 \neq 0$ in Lorentz gauge. The wave vector is of the form $(k_0, 0, 0, k_3)$, v is the wave speed k_0/k_3 , I, J run from 1 to 2, and $\tau = b_2/a_1$.

Mode	v^2	Metric Components	Aether Components
Transverse Traceless	$\frac{1}{1+\tau}$	ϵ_{IJ}	-
Transverse Vector	$\frac{2+\tau}{2+2\tau}$	$\epsilon_{0I}, \epsilon_{3I}$	w^I
Longitudinal	$\frac{2+\tau-\tau^2}{2+\tau}$	$\epsilon_{00}, \epsilon_{03}, \epsilon_{33}, \epsilon_{IJ}$	w^0, w^3

Astrophysical sources, such as coalescing black holes or neutron stars, may couple only weakly to the new modes. Even if so, a gravitational wave observatory could potentially still detect the TT modes travelling at a speed other than c by comparing time of arrival data with non-gravitational signals from the same event.

3.4 Cosmology

With a consistent gravity-aether-matter theory, one can look at the cosmological implications of Lorentz symmetry breaking. There are effects due to both the aether stress tensor and the modified field equations for matter fields. Assuming the aether frame coincides with the isotropic frame of a Robertson-Walker metric, the aether stress tensor has at most two terms. The first term is proportional to the Einstein tensor and hence renormalizes G . The second term, which is non-vanishing only if there is spatial curvature, is that of a perfect fluid with pressure equal to $-1/3$ times the energy density (like the spatial curvature term in the Friedmann equations). The aether itself therefore affects the cosmological expansion rate.

For a Lorentz-violating matter coupling like that in (1) the equation of state is different at high energies. This does not lead to dramatic consequences for cosmological evolution at energies up to k_0 ⁹. However in an inflationary scenario where cosmological scales were once much smaller than k_0 , modified dispersion could have important effects on the power spectrum of the primordial metric fluctuations (c.f. ref.¹⁰ and references therein). Results for energies above k_0 should be treated with caution though as the aether theory is a low energy effective theory and is most likely not applicable for energies at or above k_0 .

4 Viability Issues

4.1 Stability

If the aether theory is to be viable, it must be energetically stable. In a generally covariant theory the energy is given by the boundary term in the Hamiltonian; this is the ADM mass in general relativity. The energy for the aether theory with Lagrangian (4) has not yet been calculated. It may not be the ADM mass, due to the aether “kinetic term” which introduces additional $(\partial g)^2$ terms via the connection components in $\nabla_a u_b$. These terms persist even at spatial infinity because u^a is a unit vector. The usual positive energy theorem may therefore no longer apply (as it assumes that the energy is the ADM mass). What we need is a new positive energy theorem, using the correct definition of energy, to establish the range of parameters in the Lagrangian (4) for which the energy is always positive. It may be generally only positive in the asymptotic aether frame, which could be enough to guarantee stability of the theory.

4.2 Shocks

In the theory with only $a_1, b_1 \neq 0$, generic initial data will develop shocks where the integral curves of u^a cross, signaling a breakdown of the effective theory^{6,11}. The underlying source of these shocks is the insensitivity of the action to the symmetric derivative of u^a . It is hoped that including terms involving the symmetric derivative will cure this problem.

5 Conclusion

The introduction of Lorentz breaking into general relativity demands the addition of another dynamical field. Since this extra field has its own stress tensor and thereby gravitational consequences, the Lorentz breaking effects for matter are only half the story. Any new theory must be energetically stable and not contradict observations. The aether theory of a preferred frame discussed here is a candidate for such a theory, but much work remains to be done.

Work supported in part by the NSF under grant PHY98-00967.

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